

surface is continuing. Interstitial material fabrication techniques are being evaluated to identify improvements in quality and producibility. Additional tests are planned to evaluate the effects of environmental conditions (temperature, ambient pressure, shock, vibration, etc.) on the interstitial material and its thermal performance.

### Conclusions

The improved thermal joint can reduce contact resistance at metal-to-metal interfaces by up to a factor of 10. Through proper selection of materials and surface treatment, the joining surfaces may be separated easily after solidification of the low-melting-point alloy. The segmented surface can also provide low thermal resistance during high inertial accelerations. While this technology is being developed to improve thermal control of airborne electronics, it potentially can be applied to many interface heat-transfer problems where low contact resistance and ease of joint disassembly are desired.

### References

- <sup>1</sup>Mantell, C.L., ed., *Engineering Materials Handbook*, McGraw-Hill, New York, 1958.
- <sup>2</sup>Barzelay, M.E., Tong, K.N., and Holloway, G.F., "Effect of Pressure on Thermal Conductance of Contact Joints," NACA TN 3295, May 1955.

## Despin of a Liquid-Filled Cylinder Caused by Coning

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D'AMICO and Miller<sup>1</sup> reported a series of despin experiments using a laboratory spin fixture undertaken in an effort to understand the unstable flight of liquid-filled projectiles at "high viscosity" (in the sense that the Ekman number,  $E = \nu/\omega a^2$ , is larger than  $10^{-3}$ ) where  $\nu$ ,  $\omega$ , and  $a$  denote the liquid kinematic viscosity, projectile spin rate, and cavity radius. These were compared to flight data, and qualitative agreement was shown, in the limited sense that stable flights corresponded to low despin rates and unstable flights to high despin rates. No attempt to understand the mechanism was made in that report. This Note will demonstrate qualitative agreement with a mechanism drawn from the linear theory of forced coning motion.

The experiments were performed in a cylindrical container which had an inner radius  $a = 60.3$  mm and half height  $c = 258.7$  mm. The container was filled completely with a liquid of a given viscosity and spun up to 66.67 Hz about its symmetry axis, inclined 20 deg to the vertical. After the liquid-container system had come to equilibrium, the spin drive was turned off and the spin axis forced to cone at 8.33 Hz about the vertical. The spin rate was recorded as the container spun down from 66.67 to 33.33 Hz. The data were reasonably well represented by a simple exponential decay, and the decay constants were measured. The previously

measured frictional decay was subtracted, and the resulting net liquid-caused decay was converted to a (despin) moment about the spin axis.

The data can be interpreted in terms of the response of a contained rotating liquid to coning motion of its container. Let  $\Omega$  denote the coning rate,  $\alpha$  the coning angle, and  $\epsilon = (\Omega/\omega) \sin \alpha$ . If  $\epsilon$  and  $E$  (defined earlier) are both small compared to unity, the equations governing the motion of the fluid can be solved by linearizing about a state of solid rotation and applying a boundary-layer approximation. The linear response is of the order  $\epsilon$  unless the coning rate is comparable to the frequency of one of the free oscillations of the rotating liquid. In that case, the response is dominated by the resonant mode, and its amplitude is of order  $\epsilon$  divided by the sum of  $e (=E^{1/2})$  and a number representing the nearness to the resonant geometry.<sup>2</sup>

The resonant eigenfunction for the pressure is given by

$$J_1(jr) \sin(kz) \cos(\phi + \beta)$$

where  $r$ ,  $\phi$ ,  $z$  are the usual cylindrical coordinates,  $J_1$  the first-order Bessel function of the first kind, and  $j$  and  $k$  constants such that

$$\cos(kc/a) = 0; \quad sjJ_1'(j) + 2J_1(j) = 0$$

and  $j^2 = X^2 k^2$ , where

$$X^2 = (4 - s^2)/s^2; \quad s = 1 - \Omega/\omega$$

prime denotes the derivative with respect to the argument, and  $\beta$  is a phase angle which depends on viscosity.

The linear response so calculated is periodic in the azimuthal angle (measured with respect to the spin axis), and so it cannot contribute to torque about the spin axis. Thus, the despin torque must arise from the flow driven by nonlinear interactions which leads to axisymmetric torques. (In principle the linear calculation could be extended to find these torques. In practice that is an extremely intricate calculation which is well beyond the scope of this Note.) It is possible to show<sup>3</sup> that the despin torque will be dominated by the viscous shear stress in the boundary layer which has a thickness proportional to  $(\nu/\omega)^{1/2}$ . Thus, the despin torque can be written symbolically as

$$\text{torque} \propto (\text{viscosity})(\text{linear amplitude})^2 \\ \div \text{boundary-layer thickness}$$

The container used in the despin experiments just discussed has a length-to-diameter ratio of 4.291, and there are two resonances which may be important. One has a radial wavenumber of unity and an axial half-wave number of two. The other has a radial wave number of two and an axial half-wavenumber of four. The critical  $s$  and  $j$  for these modes are 0.7325281 and 2.899531 for the former and 0.840394 and 5.750889 for the latter. Thus, the critical coning rates are in the range of 1/6 to 1/4 of the rotational rate, so the despin experiments described can be expected to excite one or the other of these modes.

Interpretation of the experiment is complicated by the change in spin rate during the experiment. This leads to a continuous change in the expected critical coning rate, and a state of flow different from that postulated by the theory; i.e., the fluid cannot respond instantaneously to the change in the spin rate of its container and so is not well-modeled by a simple departure from solid rotation. In addition, it is well-known that a liquid which is spinning more rapidly than its container is likely to be centrifugally unstable.<sup>4</sup>

However, the observed rate of decay of spin is essentially constant at spin rates above about 17 Hz, and the ideas discussed earlier provide an impressive correlation with the data. Figure 1 shows the data from Ref. 1 plotted against the

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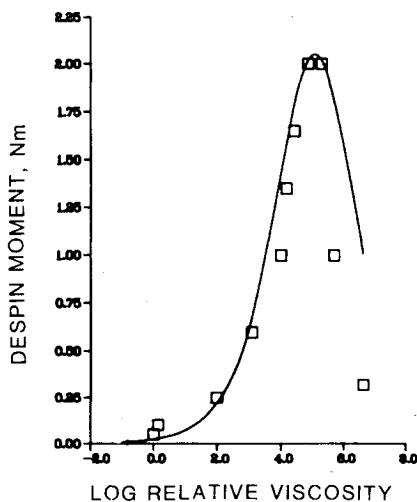


Fig. 1 Despin moment as a function of kinematic viscosity: squares denote data points and the solid line is the theoretical fit (see text).

log of the relative kinematic viscosity (with respect to that of water). The squares denote the data and the solid line is a curve of the form

$$\text{moment} = C_1 G / (C_2 + G)^2$$

where  $G = \nu^{1/2}$  and  $C_1$  and  $C_2$  are constants to be determined from experiment. A dimensional picture was chosen because the relevant nondimensional number ( $E$ ) one expects from the physics varies during an experiment. (If one were to base  $E$  on the initial rotation rate, the range corresponding to the experiment would be from  $10^{-7}$  to unity.)

The curve shown has two arbitrary constants. They were chosen by requiring that the curve and the data match at the peak, and that the peak be at the viscosity shown. No other information from the data was used, so the correlation that appears must result from the form of the fit, which was chosen from the theoretical argument outlined earlier.

The reader will note that the agreement is poor at viscosities significantly higher than that at the peak. This is reasonable, since that at the peak corresponds to an  $E$  near 0.1, above which the boundary-layer approximation is likely to be poor. Thus it seems likely that the despin process, and by extension the flight data, reflects a resonance response to the coning motion, even at relatively high viscosities. (There is a large literature regarding flight instabilities at low viscosity. A comprehensive recent bibliography is given by Murphy.<sup>5</sup>)

## References

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- <sup>2</sup>Gans, R.F., "On the Precession of a Resonant Cylinder," *Journal of Fluid Mechanics*, Vol. 41, Pt. 4, 1970, pp. 865-872.
- <sup>3</sup>Greenspan, H.P., *The Theory of Rotating Fluids*, Cambridge University Press, 1968, Chap. 1.
- <sup>4</sup>Chandrasekhar, S., *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, 1961, Chap. 7.
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## Three-Dimensional Wake Model for Low Earth Orbit

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## Nomenclature

$e$	$= 1.6 \times 10^{-19} \text{ C}$
$I$	$=$ ion current collected by a probe in wake
$I_0$	$=$ ram ion current collected by a probe
$n_i$	$=$ ambient ion density
$V_{\text{sat}}$	$=$ satellite velocity
$V_{\text{th}}$	$=$ ion thermal velocity
$\epsilon_0$	$=$ permittivity of free space
$\phi$	$=$ electrostatic potential
$\rho$	$=$ charge density
$\theta$	$=$ electron temperature

## Introduction

A CALCULATION of the electrostatic potential surrounding a model satellite which ignores ion focusing effects on space charge density is presented. The calculation is a three-dimensional simulation of a finite length right octagonal cylinder moving at Mach 8 through neutral plasma of ambient density  $n_0 = 10^5 \text{ cm}^{-3}$  and temperature  $\theta = 0.1 \text{ eV}$ . The object and the plasma parameters represent well the low-altitude AE-C satellite and its measured environment. Surface potentials on the object were obtained using the theory presented in Ref. 1. This yields a mean spacecraft potential in general agreement with that reported by Samir.<sup>2</sup> Wake ion densities were calculated assuming straight line trajectories, while electron densities were taken to be  $n_0 \exp(e\phi/\theta)$ , Ref. 3. The electrostatic potential  $\phi$  was found by solving self-consistently Poisson's equation in the space around the object. Wake potentials and ion currents predicted by this model are compared with those obtained from AE-C data by Samir. Good agreement between observation and the predictions of this relatively simple theory is found.

## Wake Potentials

Wake potentials are calculated using a preliminary version of POLAR, a computer code being developed to predict the charging of large space structures in three dimensions. The POLAR code presently consists of three major modules. The first is for object and mesh definition. The object definition is much along the lines of NASCAP<sup>4</sup> but more restrictive, there are no plans to include booms. The space around the object is broken up into discrete cells. Since the important regions are located along the flow direction, the mesh is staggered so that most of the cells are used to resolve wake and sheath phenomena.

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